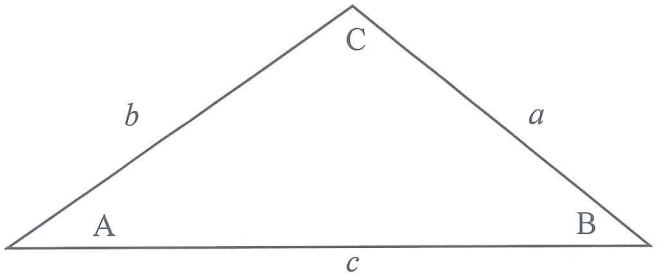
Sec. 7.6 Non Right Triangles—Law of Sines

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Use Law of Sines when you know measure of an angle and its opposite side.

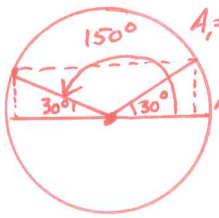
1. Two angles and any side (ASA or AAS)
2. Two sides and an angle opposite one of them (SSA – Ambiguous Case – Could have 0, 1, or 2 triangles)



There are two angles between 0° and 180° that have the same sine. This is what can create the ambiguous case using Law of Sines.

Ex: Find the two angles between 0° and 180° that have the following sines:

a. sin A = .5



$A_1 = \sin^{-1}(.5) = 30°$

$A_2 = 180 - 30 = 150°$

b. sin A = 0.3476

$A_1 = \sin^{-1}(.3476)$

$A_1 = 20.341°$

$A_2 = 180° - 20.341$

$A_2 = 159.659°$

c. sin B = 1.2358

$B_1 = \sin^{-1}(1.2358)$

NOT POSSIBLE

$-1 \le \sin B \le 1$

d. sin θ = 0.8745

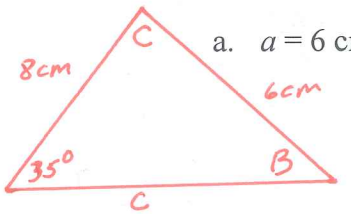
$\theta_1 = \sin^{-1}(.8745)$

$\theta_1 = 60.986°$

$\theta_2 = 180 - 60.986°$

$\theta_2 = 119.014°$

Ex: Determine how many solutions there will be and then find them if possible.



a. $a = 6$ cm, $b = 8$ cm, A = 35°

$\frac{\sin 35°}{6} = \frac{\sin B}{8}$

$\frac{6 \sin B}{6} = \frac{8 \sin 35°}{6}$

$B = \sin^{-1}\left(\frac{8 \sin 35°}{6}\right)$

$B_1 = 49.886°$

$B_2 = 180 - 49.886°$

$B_2 = 130.114°$

* Is B_2 possible? *

$35 + 130.114 < 180$

YES!

$C_1 = 180° - 35° - 49.886°$

$C_1 = 95.114°$

$C_2 = 180° - 35° - 130.114°$

$C_2 = 14.886°$

$\frac{\sin 35°}{6} = \frac{\sin 95.114°}{c_1}$

$c_1 \sin 35° = 6 \sin 95.114°$

$\frac{c_1 \sin 35°}{\sin 35°} = \frac{6 \sin 95.114°}{\sin 35°}$

$c_1 = 10.419$ cm

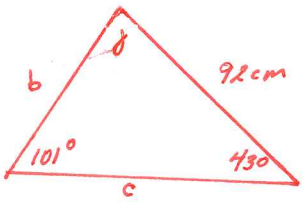
$\frac{\sin 35°}{6} = \frac{\sin 14.886°}{c_2}$

$c_2 \sin 35° = 6 \sin 14.886°$

$\frac{c_2 \sin 35°}{\sin 35°} = \frac{6 \sin 14.886°}{\sin 35°}$

$c_2 = 2.687$ cm

b. $a = 92$ cm, $\beta = 43^\circ$, $\alpha = 101^\circ$ *Note: a is opposite angle α (alpha), b is opposite angle β (beta), and c is opposite angle γ (gamma).



$\frac{\sin 101^\circ}{92} = \frac{\sin 43^\circ}{b}$

$b \sin 101^\circ = 92 \sin 43^\circ$

$\frac{b \sin 101^\circ}{\sin 101^\circ} = \frac{92 \sin 43^\circ}{\sin 101^\circ}$

$b = 63.918 \text{ cm}$

$\frac{\sin 101^\circ}{92} = \frac{\sin 36^\circ}{c}$

$c \sin 101^\circ = 92 \sin 36^\circ$

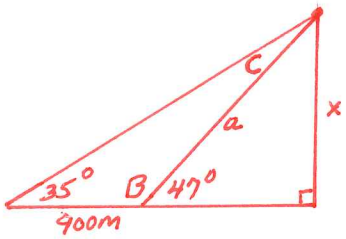
$\frac{c \sin 101^\circ}{\sin 101^\circ} = \frac{92 \sin 36^\circ}{\sin 101^\circ}$

$c = 55.088 \text{ cm}$

$\gamma = 180^\circ - 101^\circ - 43^\circ$

$\gamma = 36^\circ$

Ex: A surveyor takes two sights of the peak of a mountain at a distance 900 meters apart on a direct line to the mountain. The first observation results in an angle of elevation of 47 degrees whereas the second results in an angle of elevation of 35 degrees. If the transit is 2 meters high, what is the height h of the mountain?



$\frac{\sin 12^\circ}{900} = \frac{\sin 35^\circ}{a}$

$a \sin 12^\circ = 900 \sin 35^\circ$

$\frac{a \sin 12^\circ}{\sin 12^\circ} = \frac{900 \sin 35^\circ}{\sin 12^\circ}$

$a = 2482.875$

$\sin 47^\circ = \frac{x}{2482.875}$

$2482.875 \sin 47^\circ = x$

$1815.860 = x$

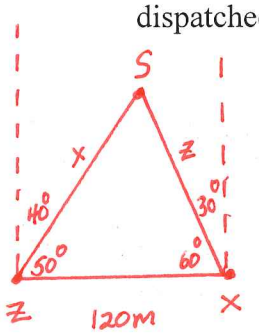
$B = 180 - 47$

$B = 133^\circ$

$C = 180 - 133 - 35$

$C = 12^\circ$

Ex: Coast Guard station Zulu is located 120 miles due west of the Station Xray. A ship at sea sends an SOS signal that is received by each station. The call to Zulu indicates that the bearing of the ship from Zulu is N40°E (40 degrees east of north) and the call to Station Xray indicates that the bearing of the ship is N30°W (30 degrees west of north). How far is each station from the ship? If a helicopter is flying 200 miles per hour is dispatched from the nearest station, how long will it take to reach the ship?



$\frac{\sin 70^\circ}{120} = \frac{\sin 60^\circ}{x}$

$x \sin 70^\circ = 120 \sin 60^\circ$

$\frac{x \sin 70^\circ}{\sin 70^\circ} = \frac{120 \sin 60^\circ}{\sin 70^\circ}$

$x = 110.593 \text{ miles}$

(STATION Z to SHIP)

$\frac{\sin 70^\circ}{120} = \frac{\sin 50^\circ}{z}$

$z \sin 70^\circ = 120 \sin 50^\circ$

$\frac{z \sin 70^\circ}{\sin 70^\circ} = \frac{120 \sin 50^\circ}{\sin 70^\circ}$

$z = 97.825 \text{ miles}$

(STATION X TO SHIP)

$d = rt$

$97.825 = 200t$

$\frac{97.825}{200} = \frac{200t}{200}$

$.489h = t$

or

29.347 min

$S = 180 - 60 - 50$

$S = 70^\circ$

Day 1 Assignment: p 305-306 #1,4,7,16,24,25,27
Day 2 Assignment: p 306-307 #30,35,36,38,44,48